Mergers of Binary Stars:
The Ultimate Heavy-Ion Experience

Madappa Prakash

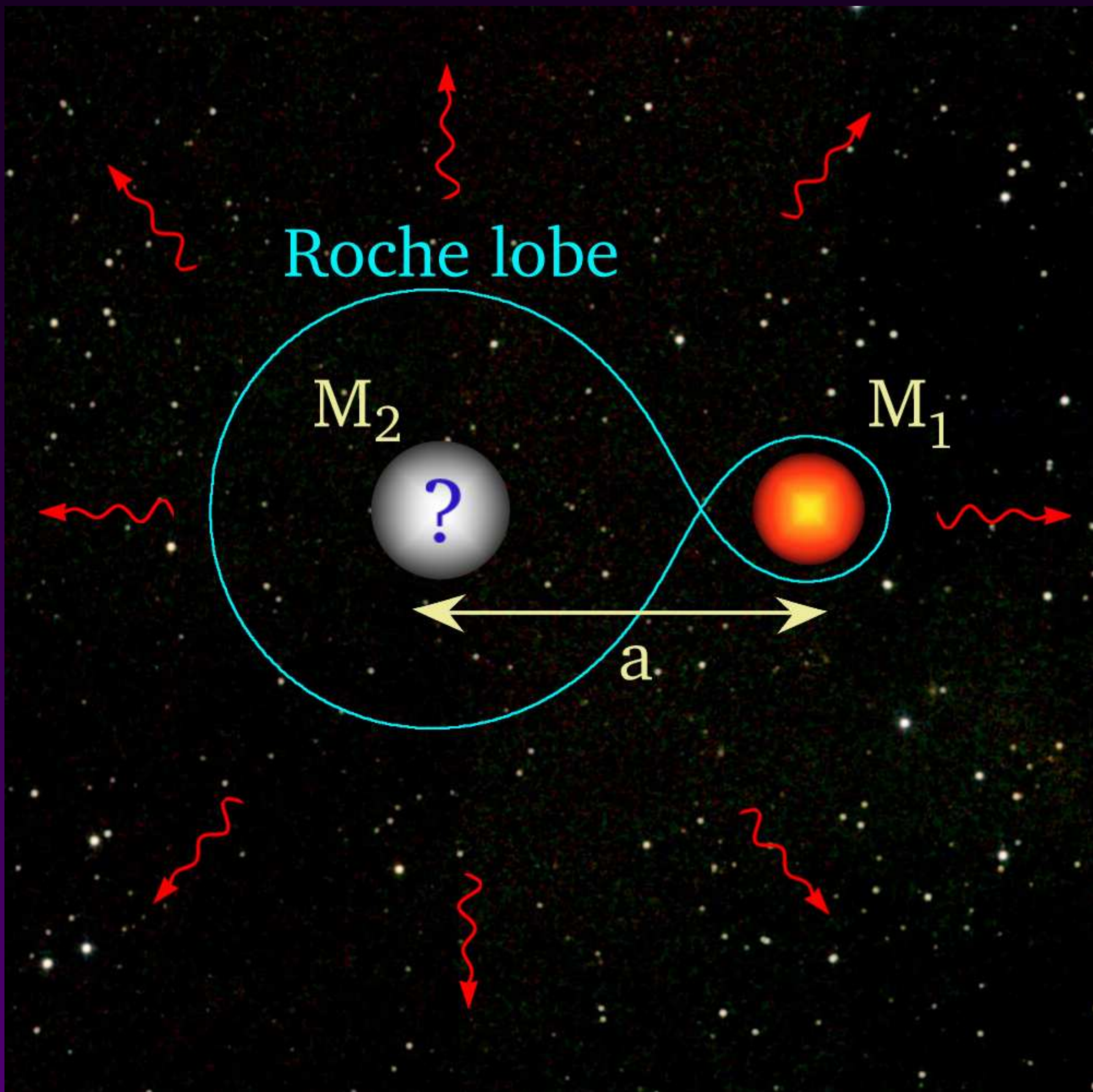
Saša Ratković

James M. Lattimer

SUNY at Stony Brook, Stony Brook, NY

QM04, Jan 11-17, Oakland

The Binary Merger Experience



- $M_1 < M_2$
- radial separation: $a(t)$
- M_1 - NS or SQM
- M_2 - BH, NS, …
- GW emission :

$$\begin{aligned} L_{GW} &= \frac{1}{5}\frac{G}{c^5}\langle \dddot{I\!\!\!-}_{jk}\dddot{I\!\!\!-}_{jk}\rangle \\ &= \frac{32}{5}\frac{G^4}{c^5}\frac{M^3\mu^2}{a^6} \end{aligned}$$

- Henceforth, whenever necessary, $G = 1$ & $c = 1$.

Merger Rates of Binary Systems

Author(s)	Information	Type	Merger Rate
Phinney (1991)	pulsar lifetimes,	cons.	5×10^{-8}
	distributions	bguess	7×10^{-6}
Van den Heuval &	pulsar detectability,	cons.	3×10^{-7}
Lorimar (1996)	distribution	bguess	8×10^{-6}
Bailes (1996)	galactic pulsar	lbound	10^{-7}
	birth rates	ubound	10^{-5}
Potegies Zwart &	"scenario machine"		$0.2 - 3$
Yungelson (1998)	w/ supernova kicks		$\times 10^{-5}$
Bethe &	common envelope	ubound	10^{-5}
Brown (1998)	hypercritical accretion		

Rates in $\mathrm{yr}^{-1}\ \mathrm{Mpc}^{-3}$

$1\ \mathrm{pc} = 3 \times 10^{18}$ cm.

Einstein's General Relativity

$$G^{\alpha\beta}\left[g, \partial g, \partial^2 g\right] = 8\pi\, T^{\alpha\beta}\left[g\right]$$

- $G^{\alpha\beta}$: 2^{nd}-order nonlinear differential operator acting on $g_{\alpha\beta}$
- $T^{\alpha\beta}$: Stress-energy tensor of matter fields

Parametrized Post-Newtonian (PPN) Formulation

In weak field limit,

$$g_{\mu\nu}^{PPN} = \eta_{\mu\nu} + h_{\mu\nu}^{1PN}(M) + h_{\mu\nu}^{2PN}(M) + h_{\mu\nu}^{3PN}(M) + \cdots$$

- $\eta_{\mu\nu}$: flat-space Minkowski metric
- M : incorporates dependence on matter fields
- $1PN, 2PN, \cdots \Rightarrow [\mathcal{O}(v^2/c^2)]^\epsilon$ with $\epsilon = 1, 2, \cdots$

For vacuum gravitational fields (in transverse traceless gauge),

$$\Box h_{\times/+} = 0$$

Gravitational Wave Detection

- GW Strain : $h(t) = F_\times h_\times(t) + F_+ h_+(t)$
  - $F_{\times,+}$: Constants of order unity
  - $h_{\times,+} \sim \frac{\delta L}{L_0} \sim \frac{1}{c^2}\,\frac{4G(E^{ns}_{kin}/c^2)}{r}$: Gravitational waveforms
    - L_0: Unperturbed length of detector arm
    - δL : Relative change in length
    - E^{ns}_{kin} : Nonspherical part of the internal kinetic energy
  - ELF : 10^{-15} - 10^{-18} Hz VLF : 10^{-7} - 10^{-9} Hz*
  - LFB : 10^{-4} Hz - 1 Hz, HFB : 1 Hz - 10^4 Hz

- Astrophysical Sources Radiating GW's in the HFB

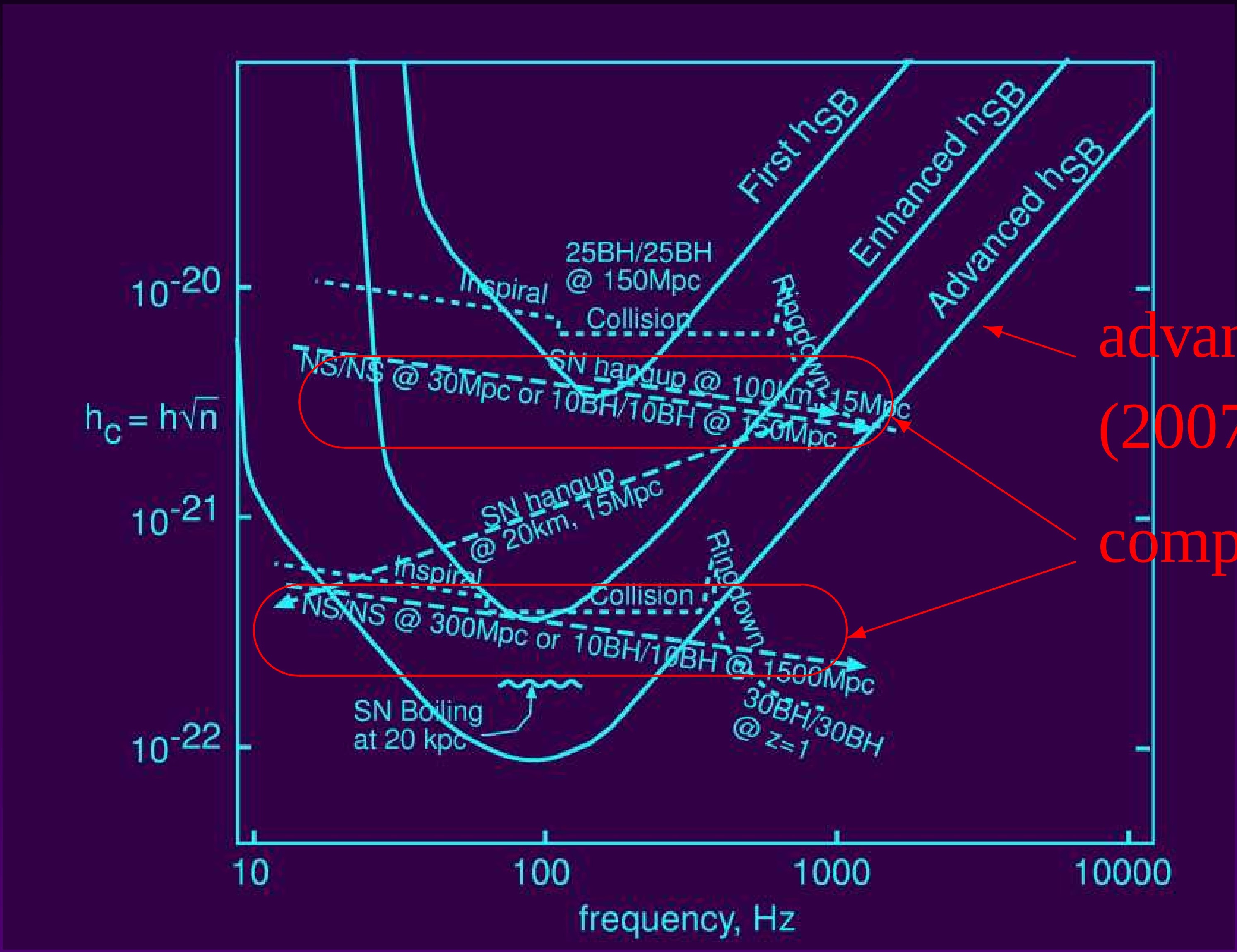
Supernovae	at 10 Mpc	$h \geq 10^{-25}$
Supernovae	Milky Way	$h \sim 10^{-18}$
$1.4M_\odot$ NS Binaries	at 10 Mpc	$h \sim 10^{-20}$
$10M_\odot$ BH Binaries	at 150 Mpc	$h \sim 10^{-20}$

GW Detectors & Expected Gains

- Ground-Based Laser Interferometers
  - LIGO, VIRGO, GEO, TAMA, ...
- The Laser Interferometer Space Antenna (LISA)

- GW's provide valuable new information "orthogonal" to electromagnetic observations
  - First direct test of GR
  - Precise ($\pm$ a few $\%$) determination of Hubble's constant H_0
  - Calibration of distance measurements
  - Masses of NS, BH (large scale structure formation)
  - ......

LIGO's Projected Sensitivity



advanced LIGO (2007?)

compact binaries

Objectives

- Explore EOS dependence of GW signals from mergers.
  - Specifically, look at differences between "normal" stars and "self-bound" (e.g., SQM) stars.
    - EOS parameter : $\alpha(M_1) \equiv d\ln(R_1)/d\ln(M_1)$
    - $\alpha_{NS} \leq 0$, while $\alpha_{SQM} \geq 0 \ (\approx 1/3)$
- Incorporate improved analysis to include GR orbital dynamics.
  - Extend the Roche lobe analysis from Newtonian to GR. GR makes stable mass transfer easier.
  - Include pseudo-GR potential to account for innermost circular orbit changes as a function of mass ratio. Has a dramatic effect on results for existence of stable mass transfer.
- Explore astrophysical consequences of differences in $\alpha(M_1)$ in (1) merger time scales and (2) GW signals.

Pseudo-GR Potentials

- Paczyński-Wiita (accretion disks)

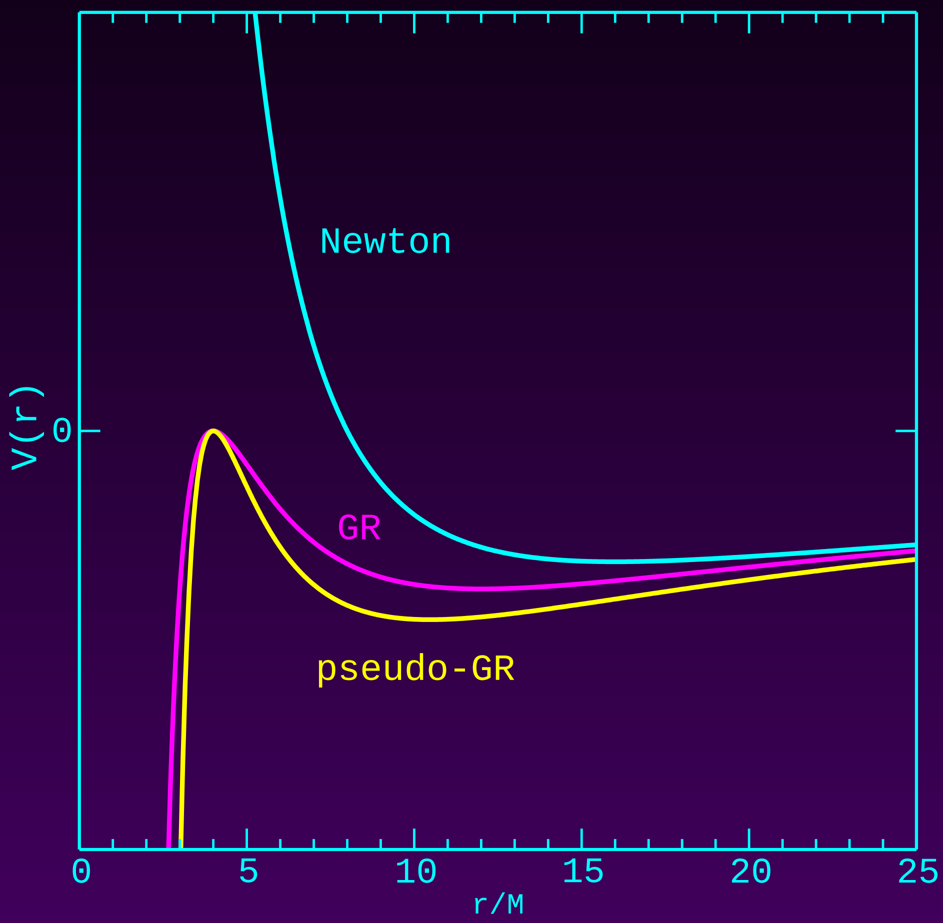
$$\phi_N(r) = -\frac{M}{r} \qquad \longrightarrow \qquad \phi_{PW}(r) = -\frac{M}{r - r_G}$$

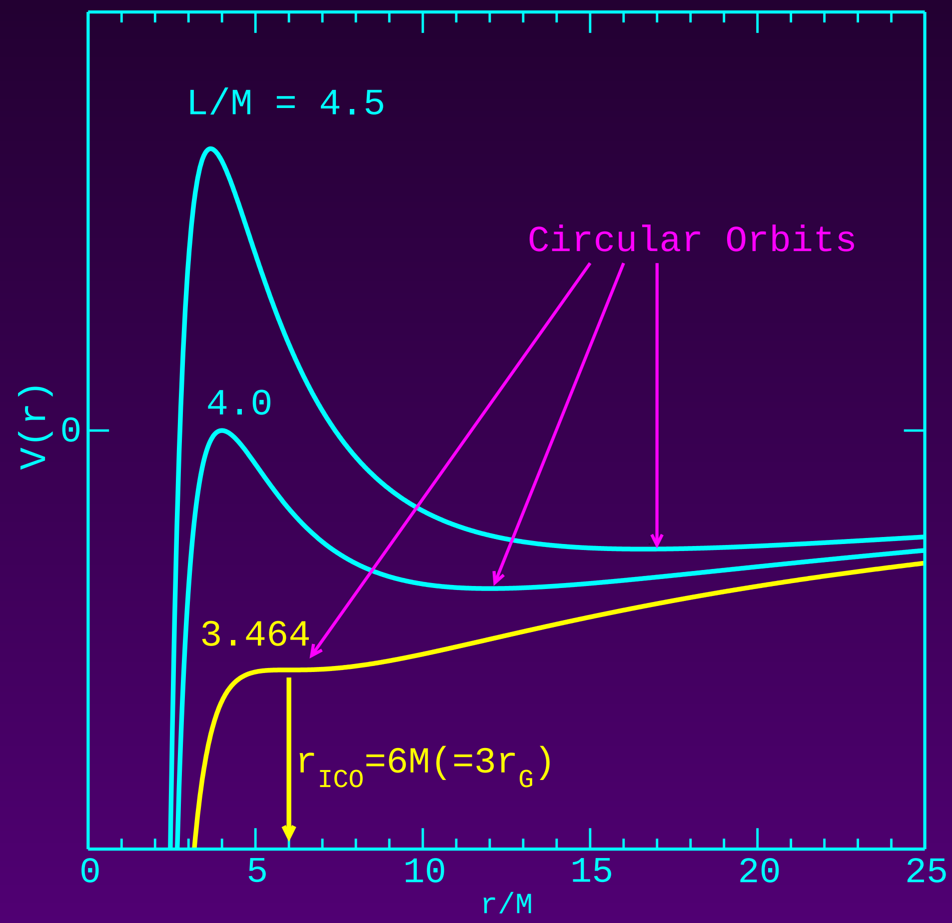
- Innermost Circular Orbit (ICO) at $r_{ICO} = 3r_G$; $r_G = 2M$
- Post-Newtonian (PN) : $r_{ICO} < 3r_G$ for $q \neq 0$
- Pseudo-GR or Hybrid Potential :

$$\phi_H(r) \;=\; -\frac{M}{r - \zeta(q) r_G}\;; \qquad q = M_1/M_2$$

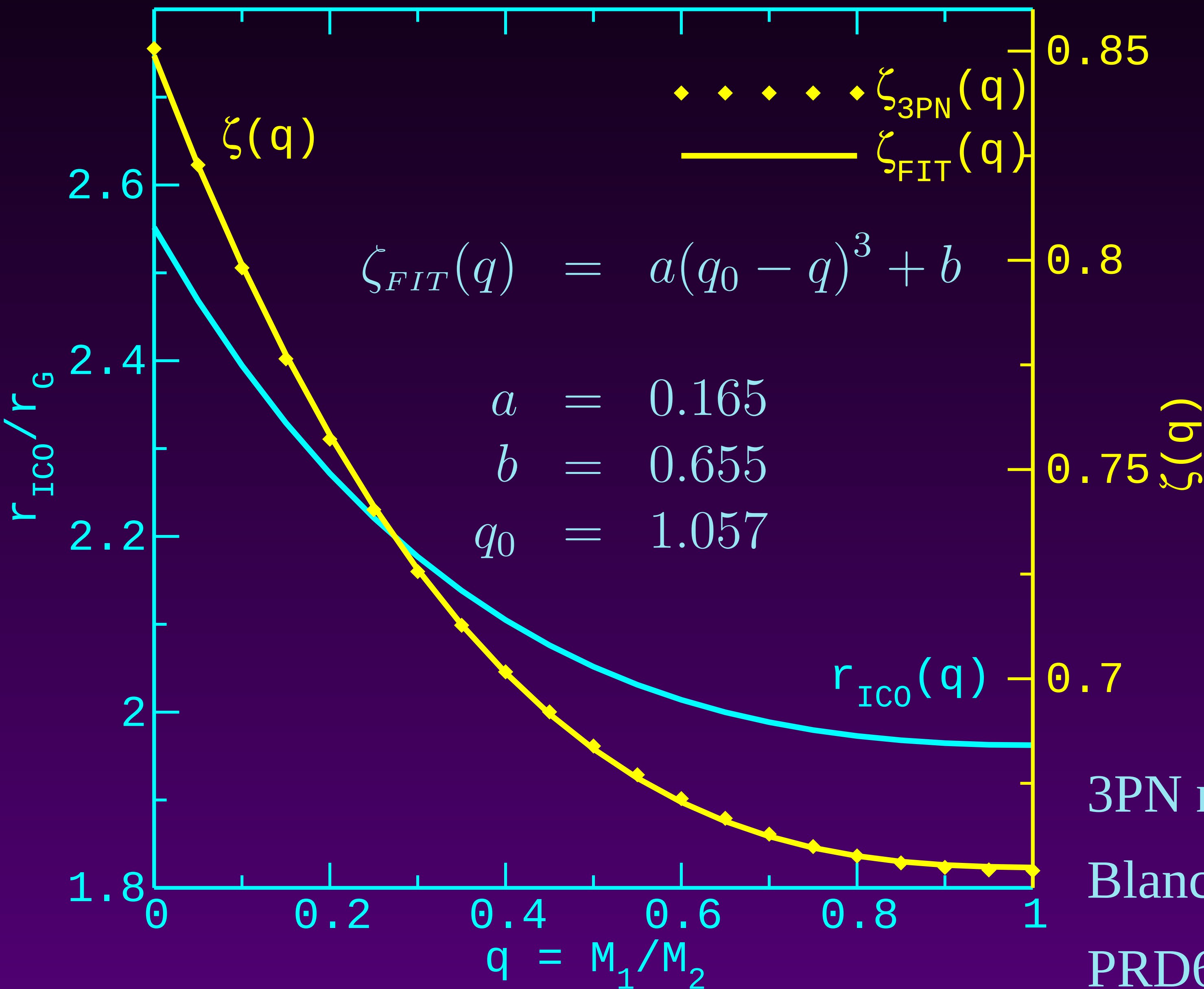
- $\zeta(q)$ - Reproduces 3PN Corrections to ICO

Effective potentials and ICO





r_{ICO} & 3PN correction factor $\zeta(q)$



3PN results from

Blanchet, D.

PRD65 (2002) 124009

Roche Lobes

- Two Rotating Bodies :

$$M_i \left(\frac{d^2 \vec{r}_i}{dt^2} \right)_{rot.} = M_i \left(\frac{d^2 \vec{r}_i}{dt^2} \right)_{in.} - M_i \vec{\omega} \times (\vec{\omega} \times \vec{r}_i) \,, \quad \omega^2 = \frac{M_1 + M_2}{a(a - \zeta(q) r_G)^2}$$

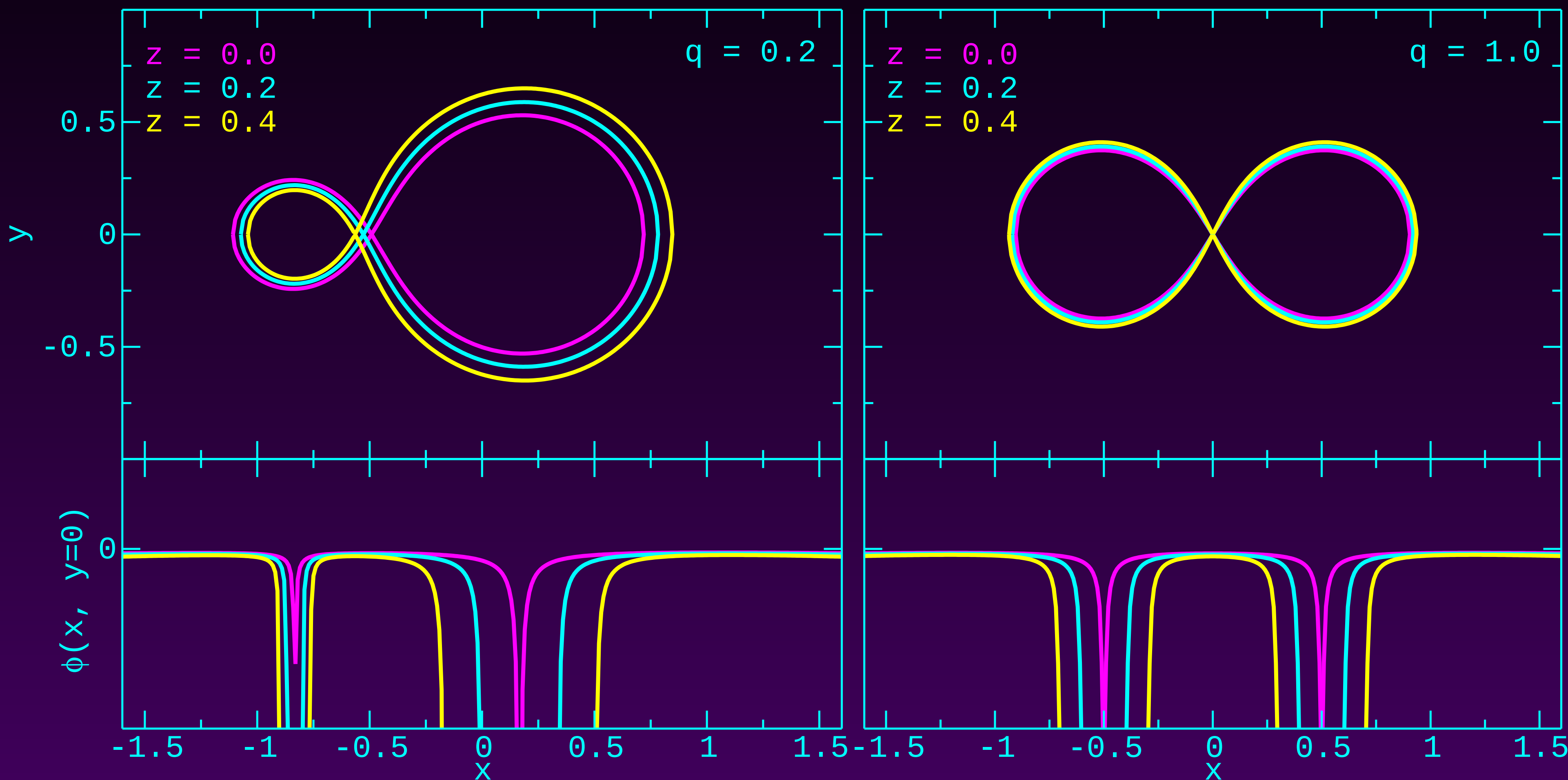
- Pseudo-GR or Hybrid Potential :

$$\phi_H^{rot}(x,y) = -\frac{M}{a} \left[\frac{x_2}{\sqrt{(x+x_1)^2 + y^2} - x_2 z} + \frac{x_1}{\sqrt{(x-x_2)^2 + y^2} - x_1 z} + \frac{1}{2} \frac{x^2 + y^2}{(1 - \zeta(q) z)^2} \right]$$

$$x = \frac{r_x}{a}, \qquad y = \frac{r_y}{a}, \qquad x_1 = \frac{1}{1+q}, \qquad x_2 = \frac{q}{1+q}$$

$$q = \frac{M_1}{M_2}, \qquad z = \frac{r_G}{a} = 2\,\frac{M_1 + M_2}{a}$$

Roche Lobes



- Effective r_{Roche} :

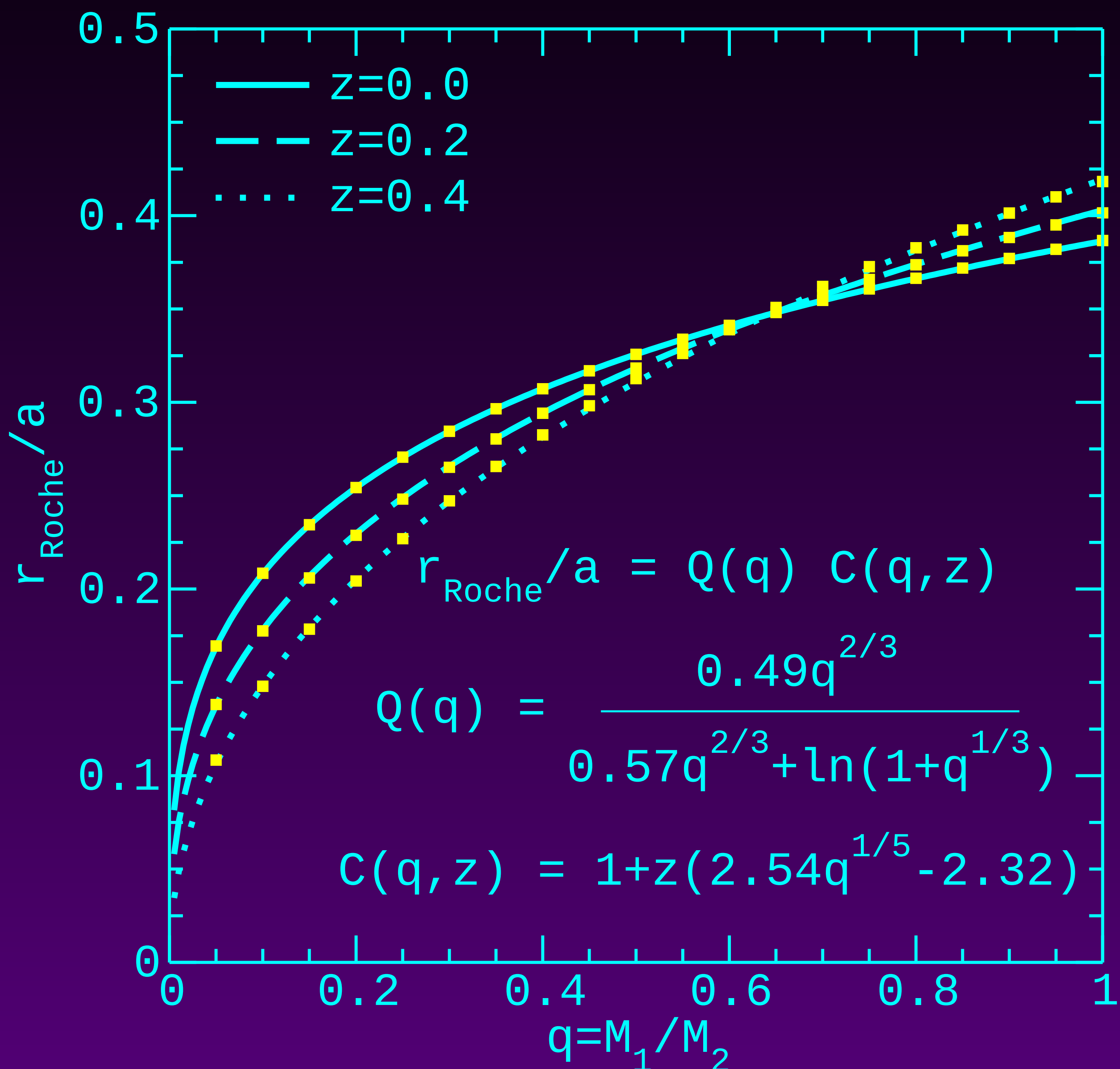
$$r_{Roche} \quad \equiv \quad \left(\frac{3}{4\pi} V_{Roche}\right)^{1/3}$$

- Dependences on q & z :

$$r_{Roche}/a = Q(q)\, C(q, z)$$

$$q = M_1/M_2 \;, \;\; z = 2\,(M_1 + M_2)/a$$

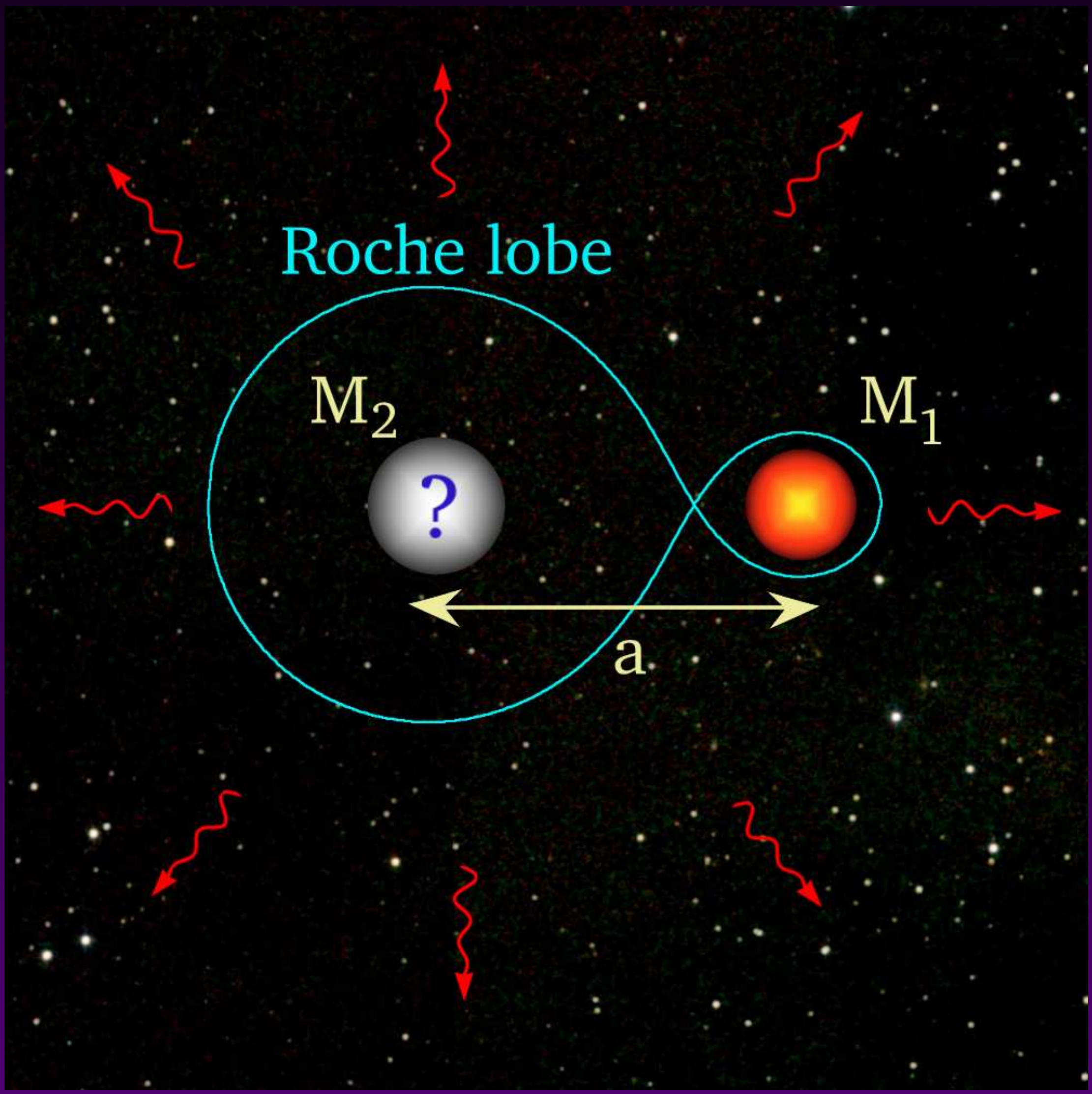
Effective Roche Radius



$$q = M_1/M_2$$

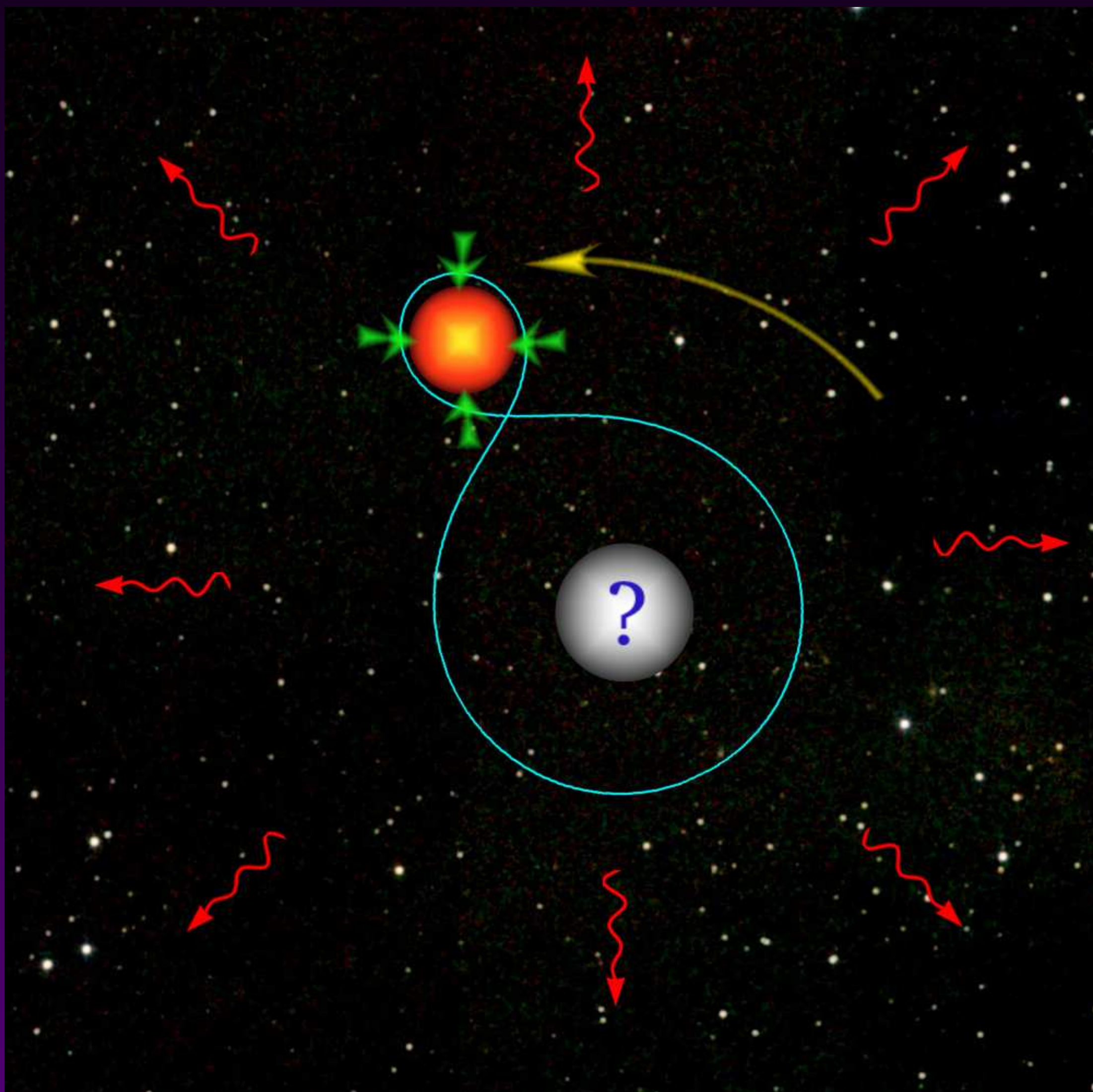
$$z = 2\,\frac{M_1 + M_2}{a}$$

Roche Lobe Overflow (1)



- $M_1 < M_2$
- Radial Separation: $a(t)$
- M_1 - NS or SQM
- M_2 - BH, NS, …
- GW Emission

Roche Lobe Overflow (2)



- Energy Loss

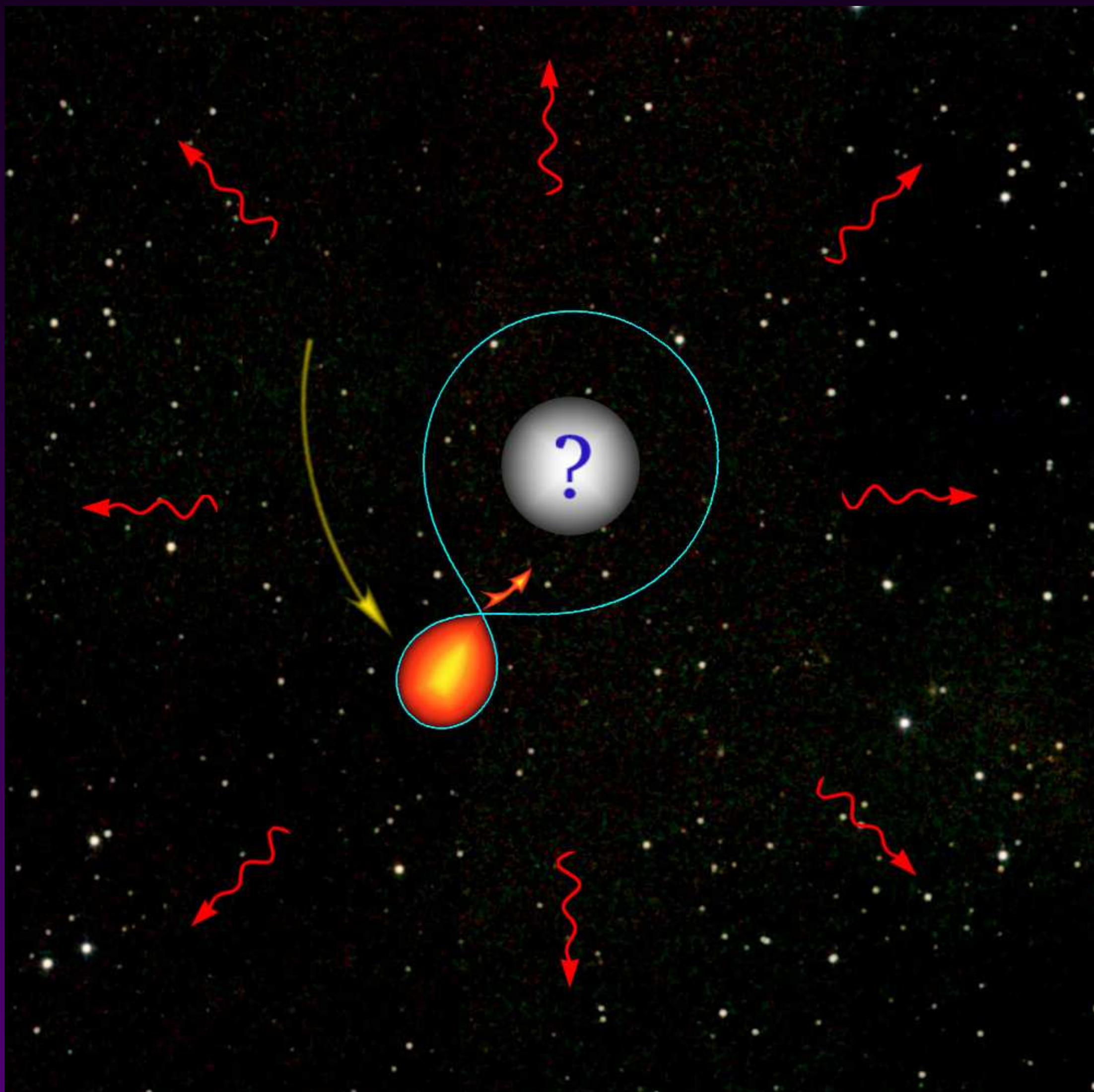
$$
\begin{aligned}
L_{GW} &= \frac{1}{5}\langle \dddot{I\!\!\!-}_{jk}\dddot{I\!\!\!-}_{jk}\rangle \\
&= \frac{32}{5}a^4\mu^2\omega^6
\end{aligned}
$$

- Angular Momentum Loss

$$
\begin{aligned}
\left(\dot{J}_{GW}\right)_i &= \frac{2}{5}\epsilon_{ijk}\langle \ddot{I\!\!\!-}_{jm}\dddot{I\!\!\!-}_{km}\rangle \\
&= \frac{32}{5}a^4\mu^2\omega^5
\end{aligned}
$$

- $a(t)$ and V_{Roche} shrink!

Roche Lobe Overflow (3)



- $R_1 = r_{Roche}$

  $\Rightarrow$ Mass transfer begins!

Orbital Evolution

- Angular Momentum Loss :

$$\left[\frac{1-q}{1+q} + \frac{r_G\, q\, \zeta'(q)}{a-\zeta(q) r_G}\right]\frac{\dot{q}}{q} + \frac{a-3\zeta(q) r_G}{2(a-\zeta(q)r_G)}\frac{\dot{a}}{a} \quad = \quad -\frac{\dot{J}_{GW}}{J_{BS}} = -\frac{32}{5}a^2\mu\omega^4$$

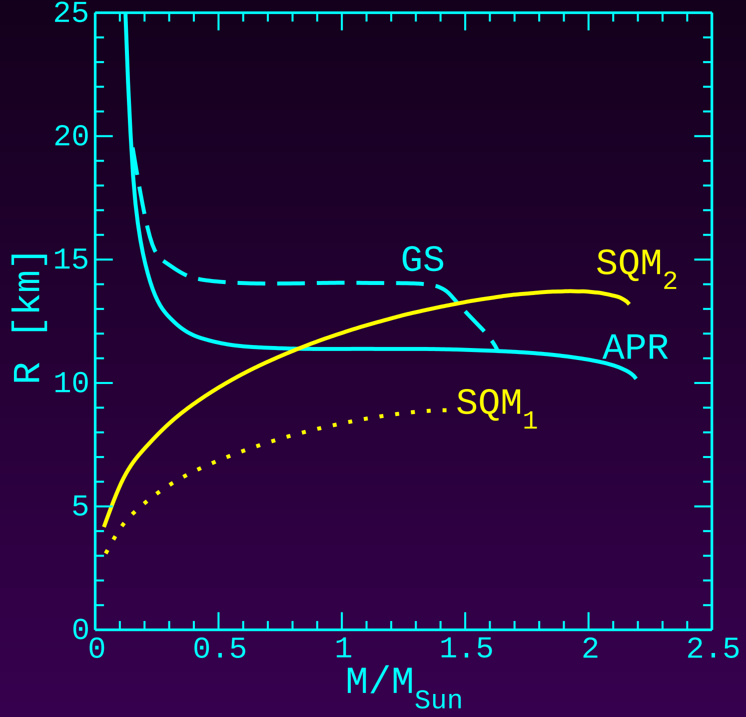
- Roche Lobe :

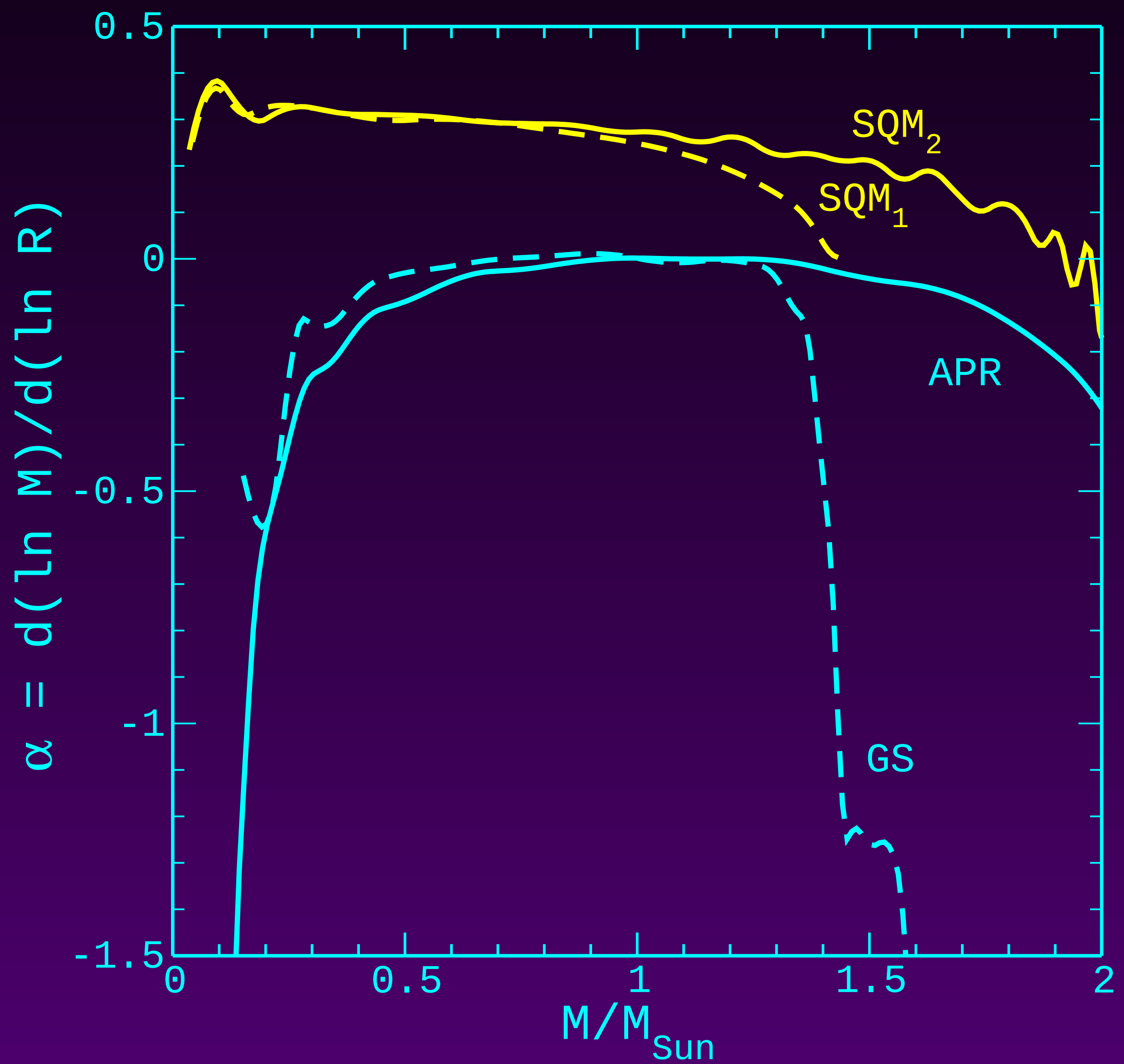
$$\frac{\dot{q}}{q} \quad = \quad \frac{1-\dfrac{\partial \ln C(q,z)}{\partial \ln z}}{\dfrac{\alpha(M_1)}{1+q} - \dfrac{\partial \ln Q(q)C(q,z)}{\partial \ln q}} \quad \times \quad \frac{\dot{a}}{a}$$

- Connection to the dense matter EOS through

$$\alpha(M_1) \quad \equiv \quad \frac{d \ln(R_1)}{d \ln(M_1)}$$

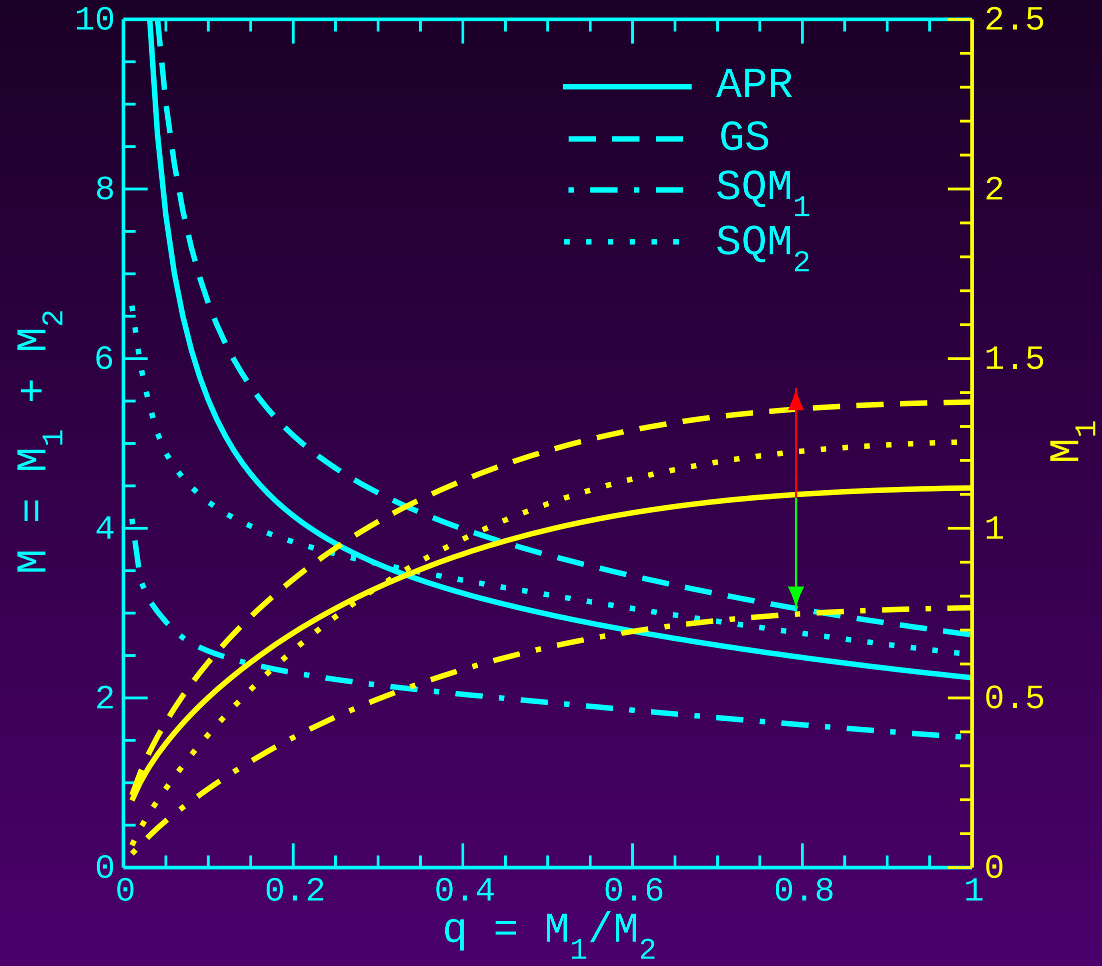
Equation of State: $\alpha(M)$





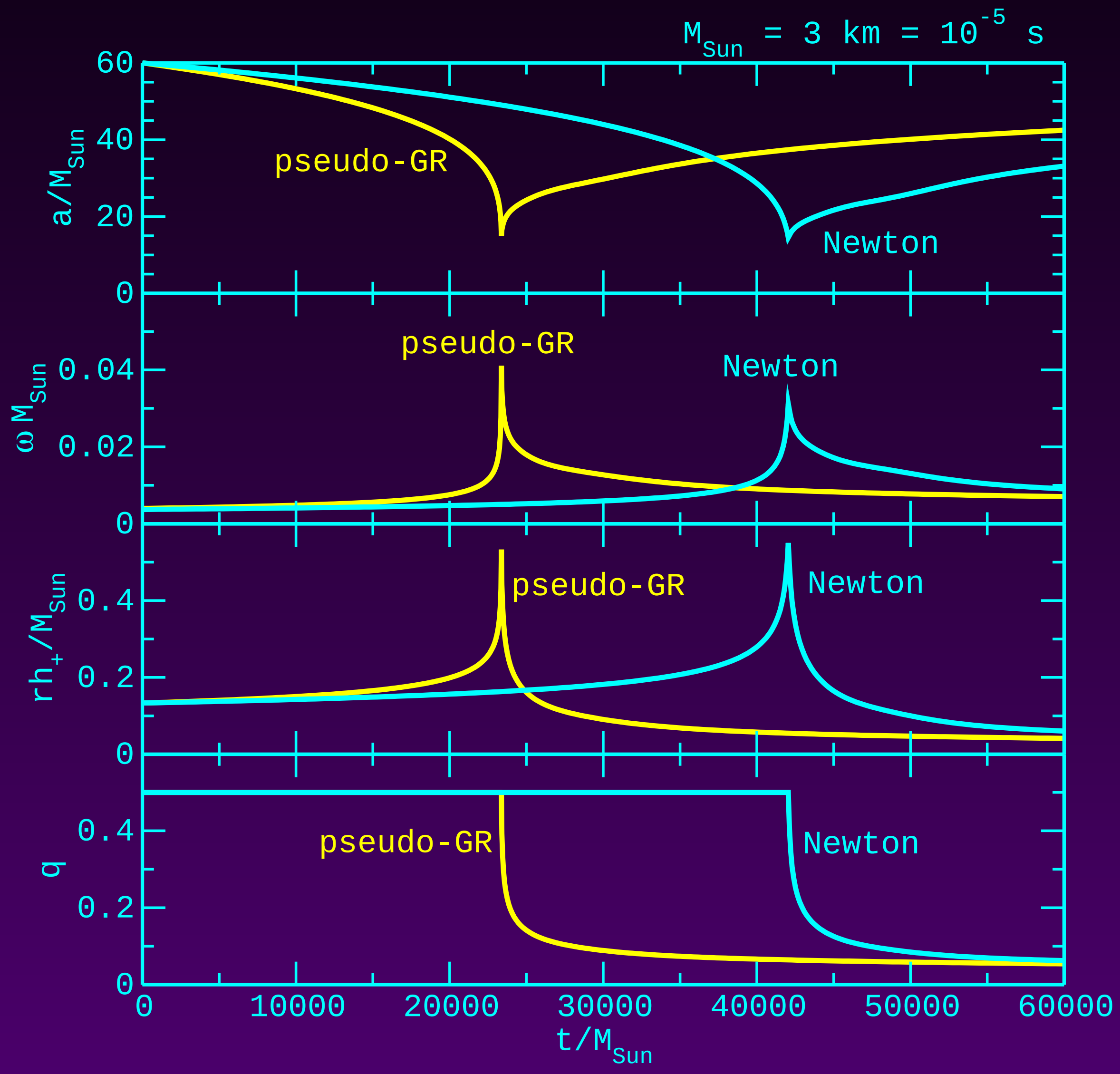
- $\alpha_{NS} \leq 0$
- $\alpha_{SQM} \geq 0$ $(\approx 1/3)$

ICO Limitations



- Mass transfer starts
  - before R_1 reaches ICO $\surd$
  - after R_1 reaches ICO $\times$
- Roche lobe filled at ICO

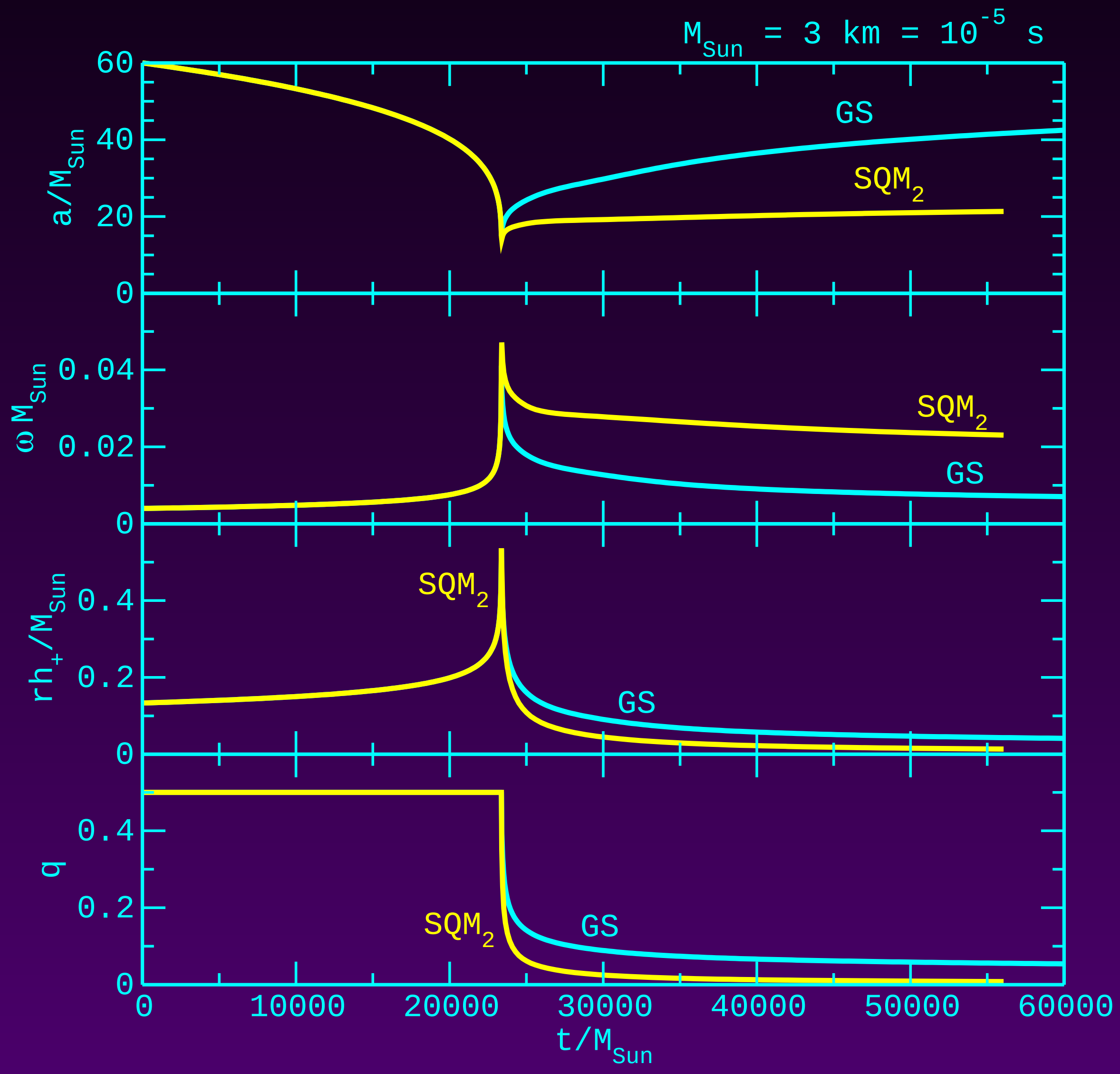
Evolution: Normal Star (GS)



$$h_+ \;=\; \frac{4}{r}\,\omega^2\,a^2\,\mu\,\cos(2\omega t)$$

- $M = 3\mathrm{M}_\odot, \quad q_{ini} = 0.5$
- GR speeds up evolution
- $a(t)$ increases after "touchdown"
- $\omega(t)$ stabilizes at long times
- Little variation among EOS's of normal stars.
- M_1 approaches the NS minimum mass; subsequent plunge (timescale $\sim$ a few minutes) yields a second spike in the GW signal!

Evolution: SQM Star



- $M = 3\mathrm{M}_{\odot}, \quad q_{ini} = 0.5$
- $a(t)$: “hovers” after “touchdown”
- $\omega(t)$: relaxes to $>> \omega_{initial}$
- $h_{+/\times}(t)$ & $q(t)$: exponential decay unlike for a NS
- $M_{1,final} \rightarrow M^{SQM}_{nugget}$ unlike for a normal star; time to tiny $M_{1,final}$ is very long!

$$h_+ \;=\; \frac{4}{r}\,\omega^2\, a^2\, \mu\, \cos(2\omega t)$$

Major Results

- Incorporating GR into orbital dynamics leads to an evolution that is faster than the Newtonian evolution.
- Large differences exist between mergers of "normal" and "self-bound (SQM)" stars.
  - SQM stars penetrate to smaller orbital radii; stable mass transfer is more difficult than for normal stars.
  - For stable mass transfer, $q = M_1/M_2$ and $M = M_1 + M_2$ limits on SQM stars are more restrictive than for normal stars.
  - The SQM case has exponentially decaying signal and mass, while normal star evolution is slower.
  - Normal stars have 2 GW peaks vs. 1 for SQM stars.

Future Tasks

- Evolution of normal & self-bound star-black hole mergers including the effects of
  - non-conservative mass transfer,
  - tidal synchronization,
  - the presence accretion disk, etc.
- Calculation of templates of expected GW signals